# Asymptotic normalization coefficient (ANC) of the system $^{13}O \rightarrow {}^{12}N + p$ determined from a ($^{12}N$,$^{13}O$) proton transfer reaction

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We report on the determination of the ANC for the system $^{13}O \rightarrow {}^{12}N + p$. The study was carried out in relation to the radiative proton capture reaction $^{12}N(p,\gamma)^{13}O$ for the role that it may play in the hot pp-chain nuclear burning processes possibly occurring in Population III stars [1]. As is generally the case with radiative proton capture reactions, due to the presence of large Coulomb barriers compared to small stellar kinetic energies, the reaction cross sections are in the order of picobarns, too small to be measured directly in the laboratory. Thus, they are studied by indirect methods. For our reaction of interest, we have applied the indirect ANC method [2] by using a ($^{12}N$,$^{13}O$) proton transfer reaction. Details of the experiment are presented elsewhere [3].

The basic of the ANC method is in the fact that the cross section for the radiative proton capture is completely determined by the Asymptotic Normalization Coefficient. This is equivalent with determining the amplitude of the tail of the overlap integral of the ground state wave function of $^{13}O$ into the two-body channel $^{12}N + p$. The ANC of the radiative capture of interest is extracted from a peripheral proton transfer reaction in which one of the two reaction vertices is characterized by the same ANC, provided that the other ANC vertex of the transfer reaction is known. In our particular case, the transfer reaction was $^{14}N(^{12}N,^{13}O)^{13}C$. Basically, the ANC of interest is extracted by fitting the experimental angular distributions with calculated Distorted Wave Born Approximation (DWBA) cross sections. This is written in the conventional DWBA formalism as:

$$\left(\frac{d\sigma}{d\Omega}\right)^{\exp}(\vartheta) = \sum_{nlj} S_{n_1 l_1 j_1}\left(^{13}O\right) S_{n_2 l_2 j_2}\left(^{14}N\right) \sigma^{DW}(\vartheta) \quad , \qquad (1)$$

where *nlj* are the usual quantum numbers that characterize in this case the proton single orbitals involved while $S_{nlj}$ represent the spectroscopic factors of the $^{13}O$ (ejectile) ground state wave function and of the $^{14}N$ (target nucleus) ground state wave function, respectively. A proton from the $^{14}N$ target occupying either the $1p_{1/2}$ or $1p_{3/2}$ orbitals is transferred to the $1p_{1/2}$ in the $^{13}O$ nucleus. Thus, using the relation $C_{nlj}^2 = S_{nlj} b_{nlj}^2$ between the asymptotic normalization coefficients of the overlap integral $C_{nlj}$, and the spectroscopic factors and the asymptotic normalization coefficients of the normalized single particle wave functions $b_{nlj}$, Eq. (1) becomes

$$\sigma_{\exp} = \left(C^{13O}_{p_{1/2}}\right)^2 \left\{ \left( \frac{C^{14N}_{p_{1/2}}}{b^{13O}_{p_{1/2}} b^{14N}_{p_{1/2}}} \right)^2 \sigma^{DW}_{p_{\frac{1}{2}} p_{\frac{1}{2}}} + \left( \frac{C^{14N}_{p_{3/2}}}{b^{13O}_{p_{1/2}} b^{14N}_{p_{3/2}}} \right)^2 \sigma^{DW}_{p_{\frac{1}{2}} p_{\frac{3}{2}}} \right\}, \qquad (2)$$

where the ANCs for the vertex $^{14}N \rightarrow {}^{13}C + p$ were determined from previous studies [4].

Before presenting the results obtained for the ANC of interest, $C_{p_{1/2}}(^{13}O)$, we briefly discuss the optical model potential (OMP) parameters used in the DWBA analysis. As one knows, the asymptotic normalization coefficients do not show a strong dependency on the proton binding nuclear potential unlike the spectroscopic factors, however the extracted value for the ANCs depend very much on the OMP parameters. These parameters are determined using measurements of elastic scattering. Instead of a typical elastic data analysis with phenomenological Woods-Saxon shaped optical potentials, we have used an analysis starting from semi-microscopic double-folding optical potentials, a procedure established at TAMU from a wide search for optical potentials to be used in the description of elastic and transfer reactions involving stable loosely bound p-shell nuclei [5]. Applying that procedure we determined the OMP parameters for the measured elastic angular distribution of the entrance channel ($^{12}N$-$^{14}N$) needed in the DWBA analysis of the transfer reaction. For the exit transfer channel ($^{13}O$-$^{13}C$) we assumed the same OMP parameters in the absence of available measured elastic data. As reported in Ref. [3], in our measurements we used a melamine $C_3H_6N_6$ target. The measured elastic angular distribution is compared in Fig. 1 (left) with predicted cross sections computed with double-folding optical potentials. We have found three sets of OMPs which describe reasonably the data for the entire angular range available in the measurements. Details of the elastic analysis are found in Ref. [6]. Here we only summarize those three sets of double-folding OMP parameters [5]:

1. $N_V = 0.37$, $N_W = 1.0$, $t_V = t_W = 1.2$ fm;
2. $N_V = 0.37$, $N_W = 0.80$, $t_V = 1.2$ fm, $t_W = 1.75$ fm;
3. $N_V = 0.37$, $N_W = 0.85$, $t_V = 1.2$ fm, $t_W = 1.75$ fm.

In order to check that we can use the same renormalization ($N_{V(W)}$) and range parameters ($t_{V(W)}$) for $^{14}N$ and $^{12}C$ target nuclei, we conducted a second experiment to measure the elastic scattering of $^{12}N$ on a pure carbon target [3]. Unfortunately, it turned out that the target used had not had the optimum thickness (6.8 mg/cm$^2$). The resulting data had not had the same quality as for the melamine target, smearing the trademark Fraunhofer oscillations, see Fig. 1 (right). Therefore, it was not reasonable to directly subtract them from the melamine data. However, we compared the experimental data on carbon with calculations made with the same potential as discussed above. More details on the results are to be found in Ref. [6].

Combing the results of both elastic measurements on melamine and carbon targets, which are in agreement with previous works [5], we could asses firmly the validity of the double-folding procedure to predict optical model potentials for the use in DWBA calculations.

Following we present the results obtained for the ANC of interest using each of the three aforementioned sets of OMPs:

1. $S^2_{p_{1/2}}(^{13}O) = 0.58$, ANC $C^2_{p_{1/2}}(^{13}O) = 2.74$ fm$^{-1}$ with $\chi^2$ (elastic) = 38.7;
2. $S^2_{p_{1/2}}(^{13}O) = 0.53$, ANC $C^2_{p_{1/2}}(^{13}O) = 2.49$ fm$^{-1}$ with $\chi^2$ (elastic) = 38.2;

3. $S^2_{p_{1/2}}(^{13}O) = 0.49$, ANC $C^2_{p_{1/2}}(^{13}O) = 2.33$ fm$^{-1}$ with $\chi^2$ (elastic) = 43.1.

Finally, we adopted an average (weighted with the chi-square values of the elastic data) of the above three values, yielding the ANC $C^2_{p_{1/2}}(^{13}O) = 2.53 \pm 0.30$ fm$^{-1}$.

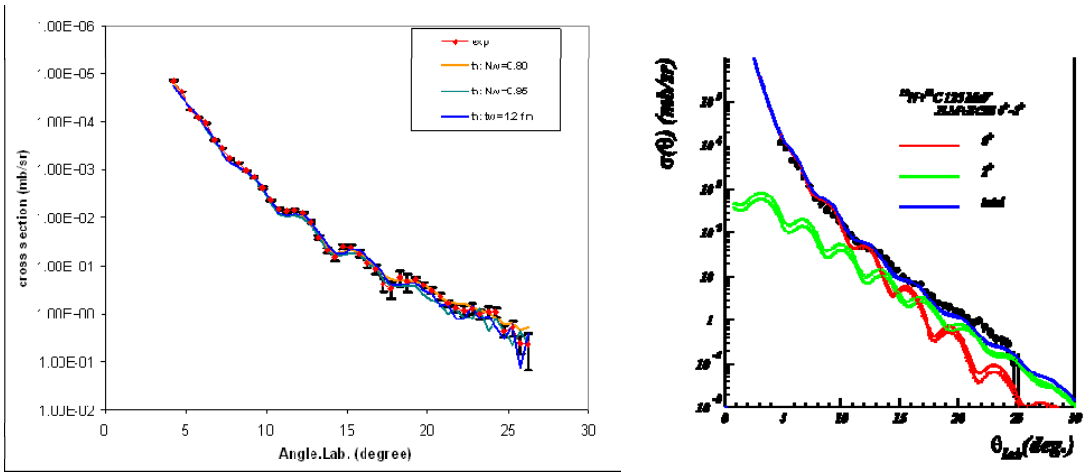


**Figure 1**. (left) Experimental angular distribution in the laboratory frame for elastic scattering of $^{12}$N projectile off $^{14}$N and $^{12}$C nuclei in the melamine target showed in comparison with predicted elastic cross section computed with three sets of double-folding optical potentials (see text for details). The theoretical calculations were smeared in a Monte Carlo simulation of the experiment that included the experimental constrains like finite size of the beam on target, beam divergence and the detector finite resolutions. (right) Experimental angular distribution in the laboratory frame for elastic scattering of $^{12}$N projectile off a 6.8 mg/cm$^2$ thick carbon target compared to theoretical calculations. The inelastic scattering to the first excited $2^+$ state in $^{12}$C could not be disentangled in the experiment from the elastic scattering. We calculated its contribution and included it in a coupled-channel calculation using the code FCIS.


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